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The Exact Linkage between the Beveridge-
Nelson Decomposition and other
Permanent-Transitory Decompositions

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The exact linkage between the Beveridge-Nelson decomposition and other permanent-transitory decompositions

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Abstract

The recent literature has raised the attention on the relationship between the Beveridge-Nelson decomposition and unobserved components processes when decomposing time series into permanent and transitory shocks. This note shows the existence of an algebraic linkage between reduced and structural forms parameters of some unobserved components processes. Results allow to reconcile standard unobserved components processes with different ARIMA models, regardless the number of structural parameters. Results are provided when the reduced forms are ARIMA(2,1,2) and ARIMA(0,2,2). For the latter, the exact relation between the Hodrick-Prescott filter and the IMA(2,2) reduced form is also shown.

Keywords: Beveridge-Nelson decomposition, Unobserved components processes, Local linear trend, ARIMA.

JEL Classification: C22, E32

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1 Introduction

The extraction of trend and cycle from a macroeconomic time series is particularly relevant for several practical reasons. In the recent years, the literature has focused the attention on the issue of using different decomposition methods leading to different trends and cycles. More specifically, Morley, Nelson and Zivot (MNZ)(2003) raise the issue that the methodology suggested by Beveridge and Nelson (1981) may lead to a trend-cycle decomposition which differs from the one obtained by using an unobserved components process (see Harvey (1989)). MNZ were able to show that this happens whenever constraints are imposed on the structural parameters of the unobserved components process. In particular, by relaxing the constraint of uncorrelated shocks (belonging respectively to the trend and cycle), the authors show that the estimated structural parameters coincide with the implied ones from the unrestricted ARIMA process representing the reduced form of the B-N decomposition¹. Similarly, Oh, Zivot and Creal (OZC)(2008) extend the analysis considering the case of different unobserved components process and therefore the case of different reduced forms (i.e. both ARIMA(2,1,2) and ARIMA(0,2,2)). Both MNZ and OZC reconcile the B-N and the unobserved components decompositions by linking the structural parameters with the autocovariances of the moving average part. Yet, the authors do not provide the exact linkages expressing the reduced form parameters as analytical functions of the structural parameters. This note fills this gap. More specifically, regardless the number of structural parameters, it is shown the existence of analytical relationships that uniquely link the reduced form parameters with those of the structural form. The remainder of this note is the following: section 2 introduces the framework and

¹Proietti (2006) has provided a clear interpretation of the UC-ARMA model with correlated shocks as the one considered by MNZ.

quickly describes the main achievements of MNZ and OZC. Section 3 contains the main (algebraic) contribution of this note making use of different reduced forms; the empirical exercises as in MNZ and OZC are reconsidered and extended. Moreover, the exact relation between the IMA(2,2) parameters and the structural parameters belonging to the Hodrick and Prescott filter are provided. Some concluding remarks are contained in section 4.

2 Motivation

This section briefly describes the two popular detrending procedures (B-N and unobserved components decompositions) together with the relation provided in MNZ and OZC. The aim of this section is to show the gap left in linking the two decompositions that will be filled in the next section.

Assuming that a univariate series y_t is an I(1) process with the following Wold representation:

$$\Delta y_t = \mu + \psi(L) = \mu + \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j} \quad (1)$$

Beveridge and Nelson (1981) have shown that the series can be decomposed into a permanent and a transitory components such that:

$$y_t = \tau_t + c_t \quad (2)$$

where the τ_t can be extracted by estimating the limiting forecast as the horizon goes to infinity, adjusted for the mean rate of growth:

$$\tau_t^{BN} = \lim_{M \rightarrow \infty} E[y_{t+M} - M\mu | \Omega_t] \quad (3)$$

Where $\Omega_t = [y_1, \dots, y_t]$ represents the information at time t . In practice, the $[\tau_t]$ series can be calculated using an unrestricted ARIMA representation for y_t . Another

option consists of using the unobserved components (UC) process as fully described in Harvey (1989). The UC benchmark model as discussed in MNZ, OZC and Proietti (2006) can be broadly defined as:

$$\begin{aligned} y_t &= \tau_t + c_t \\ \tau_t &= \tau_{t-1} + \eta_t, \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + \kappa_t + \vartheta \kappa_{t-1} \end{aligned} \quad (4)$$

In addition:

$$\text{cov} \begin{pmatrix} \eta_t \\ \kappa_t \end{pmatrix} = \begin{pmatrix} \sigma_\eta^2 & \sigma_{\eta\kappa} \\ \sigma_{\eta\kappa} & \sigma_\kappa^2 \end{pmatrix} \quad (5)$$

More specifically, the cycle is supposed to be stationary and ergodic. Moreover, as suggested by Harvey(1985), Clark(1987) and Harvey and Jaeger(1993), the cycle has two autoregressive roots in order to allow the cycle to be periodic having a peak in its spectral density function. MNZ restrict the UC model by considering $\vartheta = 0$. On the other hand, Proietti(2006) allows ϑ to be free, although he assumes that $\sigma_{\eta\kappa} = 0$. Finally OZC consider the unrestricted UC model as in (4) and (5).

Note that the reduced form of the UC model can be expressed as a restricted ARIMA process (UC-ARMA) by taking the first differences and rearranging:

$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)y_t = (1 - \phi_1 L - \phi_2 L^2)\eta_t + (1 - L)(\kappa_t + \vartheta \kappa_{t-1}) \quad (6)$$

which can be rewritten as the following ARIMA(2,1,2):

$$(1 - \phi_1 L - \phi_2 L^2)\Delta y_t = (1 + \theta_1 L + \theta_2 L^2)e_t \quad (7)$$

Given that an unrestricted ARIMA(2,1,2) can be employed in the B-N decomposition, it can be easily seen that the equivalence between the two types of decompositions relies on the moving average structure of the reduced forms (since the autoregressive part coincides).

MNZ and OZC compare the autocovariances of the moving average parts of unrestricted and restricted processes. The autocovariances functions of the unrestricted process are:

$$\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} (1 + \theta_1^2 + \theta_2^2)\sigma_e^2 \\ (\theta_1 + \theta_1\theta_2)\sigma_e^2 \\ \theta_2\sigma_e^2 \end{bmatrix}$$

To guarantee the equivalence of different decompositions, the previous autocovariances must be equal to those of the restricted UC-ARMA:

$$\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} (1 + \phi_1^2 + \phi_2^2)\sigma_\eta^2 + 2(1 - \vartheta + \vartheta^2)\sigma_\kappa^2 + 2(1 - \phi_1(\vartheta - 1) + \phi_2\vartheta)\sigma_{\eta\kappa} \\ (-\phi_1 + \phi_1\phi_2)\sigma_\eta^2 - [(1 - \vartheta)]^2\sigma_\kappa^2 + ((\vartheta - 1)\phi_1 + (1 - \phi_2)(\vartheta - 1))\sigma_{\eta\kappa} \\ (-\phi_2)\sigma_\eta^2 + (-\vartheta)\sigma_\kappa^2 + (-\phi_2 - \vartheta)\sigma_{\eta\kappa} \end{bmatrix}$$

Unfortunately this system has 3 equations and 4 unknowns parameters ($\sigma_{\eta\kappa}$, σ_κ^2 , σ_η^2 , ϑ) and therefore it allows for multiple solutions as shown in OZC. On the other hand, restricting $\vartheta = 0$ as in MNZ, the previous system admits a unique solution:

$$\begin{bmatrix} \sigma_\eta^2 \\ \sigma_\kappa^2 \\ \sigma_{\eta\kappa} \end{bmatrix} = \begin{bmatrix} (1 + \phi_1^2 + \phi_2^2) & 2 & 2(1 + \phi_1) \\ (-\phi_1 + \phi_1\phi_2) & -1 & -(1 - \phi_2 + \phi_1) \\ -\phi_2 & 0 & -\phi_2 \end{bmatrix}^{-1} \begin{bmatrix} (1 + \theta_1^2 + \theta_2^2)\sigma_e^2 \\ (\theta_1 + \theta_1\theta_2)\sigma_e^2 \\ \theta_2\sigma_e^2 \end{bmatrix}$$

Clearly this result permits to compare the “implied” estimates for the UC process given the knowledge of the unrestricted ARIMA process with those of the estimated ones (through the Kalman filter). This is useful since, as in the case of the US Gross Domestic Product, MNZ are able to show that implied and estimated parameters are very similar².

Yet, note that we know little on the reverse relation given that the MA parameters are non linear functions of the structural shocks. In other words, at this stage we are not able to make the same exercise comparing implied and estimated ARIMA parameters

²Recently, Oh and Zivot(2006) extend the results as in MNZ to the Clark model having a more complex reduced form.

based on the Kalman filter parameters. This can be considered redundant in the case of exact identification as in MNZ. However, this might be very useful if we consider the case of over-identification as in OZC. That is, when the number of structural parameters is higher than the number of the autocovariances of the moving average part of (7). In this case, based on the unrestricted ARIMA estimates, multiple admissible solutions are available for the structural parameters and therefore we lose control of the structural parameters unless further restrictions are imposed. On the contrary, comparing estimated ARIMA parameters with the implied ones, based on the Kalman filter, allows univocally to check possible discrepancies between different decomposition procedures. This point is reinforced by Harvey and Todd(1983), Harvey(1985) and Harvey and Jaeger(1993) who argue that the structural UC model can evaluate the business cycle dynamics better than unrestricted ARIMA models. Yet, as observed in OZC, *“an appropriate ARIMA model can capture the same type of dynamic behavior as a structural UC-ARMA model”*. This emphasizes the importance of the reverse relation (i.e. UC structural parameters $\rightarrow$ UC-ARMA parameters). This relation is provided in the next section and it fills the gap left in the discussion on the linkages between UC and BN decompositions.

3 Results

It is well known that the parameters of an MA(2) process are nonlinear functions of the autocovariances (see for example Box, Jenkins and Reinsel (2008) and Stralkowski, Wu and Devor (1974)). Yet, in this section we show an algebraic procedure that allows to express the three moving average parameters as exact functions of the structural ones. The starting point of the analysis is the system expressing the three autocorrelation of the

right hand side of (7) as function of the moving average parameters (θ_1 , θ_2 and σ_e^2):

$$\frac{\gamma_2}{\gamma_0} = \frac{\theta_2}{(1+\theta_1^2+\theta_2^2)}$$

$$\frac{\gamma_1}{\gamma_0} = \frac{\theta_1+\theta_1\theta_2}{(1+\theta_1^2+\theta_2^2)}$$

$$\sigma_e^2 = \frac{\gamma_0}{(1+\theta_1^2+\theta_2^2)}$$

These equations represent respectively the second and first order autocorrelations of the MA process and the variance of e_t .

From the previous system it is easy to derive the following analytical solutions:

$$\theta_2 = \frac{\gamma_2}{\sigma_e^2}$$

$$\theta_1 = \frac{\gamma_1}{(\sigma_e^2+\gamma_2)}$$

In addition, substituting these solutions in the last equation of the system, the following quartic equation in x (with $x = \sigma_e^2$) can be obtained:

$$\frac{x^4 + (2\gamma_2 - \gamma_0)x^3 + (2\gamma_2^2 - 2\gamma_2\gamma_0 + \gamma_1^2)x^2 + (2\gamma_2^3 - \gamma_2^2\gamma_0)x + \gamma_2^4}{x(x + \gamma_2)^2} = 0$$

This equation has four different solutions. Yet, the only solution leading to the invertible process³ is:

$$\sigma_e^2 = \frac{1}{4} \left(\gamma_0 - 2\gamma_2 + G + \sqrt{2} \sqrt{\gamma_0^2 + \gamma_0 G - 2(\gamma_1^2 + \gamma_2(2\gamma_2 + G))} \right) \quad (8)$$

with:

$$G = \sqrt{(\gamma_0 - 2\gamma_1 + 2\gamma_2)(\gamma_0 + 2\gamma_1 + 2\gamma_2)} \quad (9)$$

and:

$$\begin{aligned} \theta_2 &= \frac{4\gamma_2}{\gamma_0 - 2\gamma_2 + G + \sqrt{2} \sqrt{\gamma_0^2 + \gamma_0 G - 2(\gamma_1^2 + \gamma_2(2\gamma_2 + G))}} \\ \theta_1 &= \frac{4\gamma_1}{\gamma_0 + 2\gamma_2 + G + \sqrt{2} \sqrt{\gamma_0^2 + \gamma_0 G - 2(\gamma_1^2 + \gamma_2(2\gamma_2 + G))}} \end{aligned} \quad (10)$$

³A process with roots of the characteristic function that lie outside the unit circle (i.e. $\theta_2 + \theta_1 < 1$; $\theta_2 - \theta_1 < 1$; $-1 < \theta_2 < 1$)

Where γ_0, γ_1 and γ_2 are simple expressions of the structural parameters. Here we present an empirical application that makes use of the empirical results as obtained in MNZ and OZC. Both papers focus on the U.S. real GDP but they employ different sample sizes. As an exercise, we calculate the implied reduced form estimates derived from the knowledge of the estimated UC models. Table 1 refers to the results as in MNZ while Table 2 refers to OZC. Tables contain the UC estimated parameters, the ARMA parameters as they appear in the papers and the implied ARMA parameters obtained by plugging the structural parameters into (8) and (10).

Table 1. Estimated and Implied ARMA parameters (MNZ)

	UC-0		Estimated	Implied-UC-0
ϕ_1	1.530	ϕ_1	1.342	1.530
ϕ_2	-0.609	ϕ_2	-0.706	-0.609
σ_η^2	0.689	θ_1	-1.054	-1.247
σ_κ^2	0.619	θ_2	0.518	0.303
$\sigma_{\eta\kappa}$	0.000	σ_e	0.9694	0.978

Table 2. Estimated and Implied ARMA parameters (OZC)

	UC-0	Proietti		Estimated	Implied-UC-0	Implied-Proietti
ϕ_1	1.497	1.485	ϕ_1	1.365	1.497	1.485
ϕ_2	-0.569	-0.564	ϕ_2	-0.782	-0.569	-0.564
σ_η^2	0.359	0.450	θ_1	-1.110	-1.200	-1.181
σ_κ^2	0.396	0.268	θ_2	0.622	0.246	0.238
$\sigma_{\eta\kappa}$	0	0	σ_e	0.905	0.911	0.911
ϑ	0	0.207				

Note that MNZ used an UC model that restricts $\sigma_{\eta\kappa}$ to zero only (i.e. UC-0), while OZC consider also the Proietti(2006) procedure. It can be clearly seen that the restrictions imposed have an evident impact on the ARMA parameters. This is especially true in Table 2 where the implied θ_2 is nearly three times lower than the estimated. Interestingly, the Proietti and UC-0 forms report very similar implied estimates. Therefore, as already noted in MNZ, it seems that the most problematic cause of divergence between

the structural and reduced forms is the restriction $\sigma_{\eta\kappa} = 0$. Indeed, as also shown in OZC, by relaxing this assumption the unrestricted and restricted ARMA parameters match⁴.

Similar relations can be shown whenever the univariate series y_t is an I(2) process. For example, consider the standard local linear trend model:

$$\begin{aligned} y_t &= \tau_t + \kappa_t \\ \tau_t &= \tau_{t-1} + d_{t-1} + \eta_t, \\ d_t &= d_{t-1} + \xi_t \end{aligned} \tag{11}$$

where:

$$\text{cov} \begin{pmatrix} \kappa_t \\ \eta_t \\ \xi_t \end{pmatrix} = \begin{pmatrix} \sigma_\kappa^2 & \sigma_{\eta\kappa} & \sigma_{\kappa\xi} \\ \sigma_{\eta\kappa} & \sigma_\eta^2 & \sigma_{\eta\xi} \\ \sigma_{\eta\xi} & \sigma_{\kappa\xi} & \sigma_\xi^2 \end{pmatrix} \tag{12}$$

whose reduced form can be expressed as an ARIMA(0,2,2):

$$(1-L)^2 y_t = \xi_{t-1} + (1-L)\eta_t + (1-L)^2 \kappa_t \tag{13}$$

The UC-ARMA parameters be compared with the unrestricted ARIMA(0,2,2):

$$(1-L)^2 y_t = (1 + \theta_1 L + \theta_2 L^2) e_t \tag{14}$$

In particular, the exact linkage between the structural and reduced form parameters can be obtained using (8)-(10) and considering that:

$$\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 2\sigma_\eta^2 + \sigma_\xi^2 + 6\sigma_\kappa^2 + 2\sigma_{\eta\xi} + 6\sigma_{\eta\kappa} + 2\sigma_{\kappa\xi} \\ -\sigma_\eta^2 - 4\sigma_\kappa^2 - \sigma_{\eta\xi} - 4\sigma_{\eta\kappa} - 2\sigma_{\kappa\xi} \\ \sigma_\kappa^2 + \sigma_{\eta\kappa} + \sigma_{\kappa\xi} \end{bmatrix}$$

Clearly, the number of parameters of the local linear trend is higher than that of the reduced form and so the (ARIMA→UC) relation provided in OZC does not produce a unique solution for the structural parameters unless further restrictions are imposed. For example, if we consider the full orthogonal local linear trend model, then the three structural

⁴the estimates obtained by relaxing $\sigma_{\eta\kappa} = 0$ can be found in MNZ (see their Table 4) and OZC (see their Table 2).

variances are uniquely identified by the moving average parameters. Yet, the drawbacks of restricting the structural form are now clear. On the other hand, the reconciliation between alternative decompositions may be achieved by simply using the algebraic linkages. To conclude, no matter the number of parameters in the UC process, it is always possible to reconcile structural and reduced forms.

3.1 The Hodrick-Prescott filter

Based on the results achieved in the previous section it is possible to derive the linkages between the structural and reduced forms of other popular decompositions. The Hodrick and Prescott(1997) decomposition (H-P filter) represents probably the most popular procedure used by economists to extract the trend from macroeconomic time series. It is well known that the H-P filter can be considered as a specific case of the local linear trend as in (11) by considering orthogonal shocks and restricting the variance of the shock of the trend to be zero (i.e. $\sigma_\eta = 0$). It is easily seen that the autocovariance functions of this peculiar local linear trend are:

$$\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \sigma_\xi^2 + 6\sigma_\kappa^2 \\ -4\sigma_\kappa^2 \\ \sigma_\kappa^2 \end{bmatrix}$$

Therefore, the reduced form of this smooth trend model is an IMA(2,2) whose parameters can be expressed as exact functions of the structural ones such that:

$$\begin{bmatrix} \theta_2 \\ \theta_1 \\ \sigma_e^2 \end{bmatrix} = \begin{bmatrix} \frac{4\sigma_\kappa^2}{(4\sigma_\kappa^2 + \sigma_\xi(\sigma_\xi + \sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2}) + \sqrt{2\sigma_\xi(\sigma_\xi^3 + 12\sigma_\xi\sigma_\kappa^2 + \sigma_\xi^2\sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2} + 4\sigma_\kappa^2\sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2})} \\ \frac{-16\sigma_\kappa^2}{(8\sigma_\kappa^2 + \sigma_\xi(\sigma_\xi + \sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2}) + \sqrt{2\sigma_\xi(\sigma_\xi^3 + 12\sigma_\xi\sigma_\kappa^2 + \sigma_\xi^2\sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2} + 4\sigma_\kappa^2\sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2})} \\ \frac{1}{4}(4\sigma_\kappa^2 + \sigma_\xi(\sigma_\xi + \sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2}) + \sqrt{2\sigma_\xi(\sigma_\xi^3 + 12\sigma_\xi\sigma_\kappa^2 + \sigma_\xi^2\sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2} + 4\sigma_\kappa^2\sqrt{\sigma_\xi^2 + 16\sigma_\kappa^2})} \end{bmatrix} \quad (15)$$

Note that the H-P filter assumes that $\lambda = \frac{\sigma_\kappa^2}{\sigma_\xi^2}$ where λ is generally imposed to be equal to 100, 1600 and 14400 for respectively annual, quarterly and monthly data. Such that the H-P filter removes a smooth trend from some given data y_t as follows:

$$\min_{\tau_1, \dots, \tau_T} \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=1}^T ((1-L)^2 \tau_t)^2$$

Considering the mentioned generally accepted assumptions on λ and substituting respectively $\sigma_\xi^2 = \frac{\sigma_\kappa^2}{100}$; $\sigma_\xi^2 = \frac{\sigma_\kappa^2}{1600}$ and $\sigma_\xi^2 = \frac{\sigma_\kappa^2}{14400}$ into (15), the three reduced form parameters collapse to:

$$\begin{aligned} \begin{bmatrix} \theta_2 \\ \theta_1 \\ \sigma_e^2 \end{bmatrix} &= \begin{bmatrix} 0.638231 \\ -1.55834 \\ \frac{\sigma_\kappa^2 [401 + \sqrt{1601} + \sqrt{2402 + 802\sqrt{1601}}]}{400} \end{bmatrix} \\ \begin{bmatrix} \theta_2 \\ \theta_1 \\ \sigma_e^2 \end{bmatrix} &= \begin{bmatrix} 0.799444 \\ -1.77709 \\ \frac{\sigma_\kappa^2 [6401 + \sqrt{25601} + \sqrt{19201 + 6401\sqrt{25601}}]}{6400} \end{bmatrix} \\ \begin{bmatrix} \theta_2 \\ \theta_1 \\ \sigma_e^2 \end{bmatrix} &= \begin{bmatrix} 0.878847 \\ -1.87103 \\ \frac{\sigma_\kappa^2 [57601 + \sqrt{230401} + \sqrt{172801 + 57601\sqrt{230401}}]}{57600} \end{bmatrix} \end{aligned}$$

Similar results for the quarterly case were previously achieved by Kaiser and Maravall (2001), Maravall and Del Rio(2007) and in the empirical analysis of OZC (see Table 3). It can be seen that, the higher the value assigned to λ the more the roots of the moving average parameters get close to the non-invertability region.

An empirical example would help clarify this issue. Zarnowitz and Ozyildirim (2006) employ a local linear trend to decompose permanent and transitory components from the monthly natural log of the U.S. Coincident Indicator from 1948 to 2000. The authors make use of the local linear trend as in (11) by considering orthogonal shocks and restricting the variance of the shock of the trend to be zero. In particular, they claim “*the variance*

of the residuals in the equation for the trend slope should be small, and so we assume $\sigma_\xi^2 = 10^{-9}$ while fixing σ_η^2 at zero. The estimated value of σ_ϵ^2 for the natural log of the U.S. CI for 1948 2000 is 0.0005". Therefore, the authors impose two restrictions out of the three parameters. These lead to $\lambda = \frac{\sigma_\kappa^2}{\sigma_\xi^2} = 500000$ (much higher than 14000 as suggested by the H-P filter)⁵. In addition, the reader can easily see that substituting the imposed $\sigma_\xi^2 = 10^{-9}$ and estimated $\sigma_\epsilon^2 = 0.0005$ in (15) we obtain the following IMA(2,2) parameters: $\theta_1 = -1.947$; $\theta_2 = 0.948$ and $\sigma_e = 0.0229$ whose roots are very close to the non invertibility regions. Therefore, as already noted by MNZ, imposing restrictions on the structural parameters might lead to unpleasant results.

4 Conclusions

This note has established the exact relations linking the reduced form parameters to the structural ones. That is, provided that the reduced form can be expressed as an ARIMA(p,d,q) with $q \leq 2$, no matter the number of structural parameters, it is always possible to express the moving average parameters as algebraic functions of those belonging to the generation process. The relation between parameters is also shown when the Hodrick-Prescott filter is assumed as a detrending option. One of the limitation of the algebraic result is that the exact linkage cannot be provided when the moving average part has more than 3 parameters (i.e. $q > 2$). Yet, in general, as discussed by several authors, a parsimonious ARMA process is generally sufficient to catch the business cycle properties of macroeconomic time series.

⁵For the sake of clarity please note that the variance of the noise σ_ϵ^2 in Zarnowitz and Ozyildirim (2006) corresponds to σ_κ^2 in this paper.

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